

# Exercises for Thanksgiving

November 19, 2008

Here are several questions to try your skills.

1. Construct a model for an amalgamated product as a group of permutations. Use this construction to prove that HNN extensions exist.
2. Read the section in the notes I have provided to prove that if  $F$  is a free group on  $\{a, b\}$ , then the derived group  $F'$  of  $F$  is freely generated by the elements

$$[a^i, b^j] (i, j \in \mathbb{Z}, ij \neq 0).$$

3. If  $a, b, s, t$  are elements of a group and if

$$[a, b] = [s, t] = 1, [a, a^s] = 1, a^t = aa^s$$

prove that

$$[a, a^{s^i}] = 1 (i \in \mathbb{Z}).$$

4. Construct two non-abelian, non-isomorphic semi-direct products of a free abelian group of rank two by an infinite cyclic group.